Using Decay Angle Correlations to Detect CP Violation in the Neutral Higgs Sector

B. Grządkowski

Institute of Theoretical Physics
Department of Physics, Warsaw University, Warsaw, PL-00-681 Poland

J.F. Gunion

Davis Institute for High Energy Physics
Department of Physics, University of California, Davis, CA 95616 U.S.A.

Abstract

We demonstrate that decay angle correlations in $\tau^-\tau^+$ and $t\bar{t}$ decay modes could allow a determination of whether or not a neutral Higgs boson is a CP eigenstate. Sensitivity of the correlations is illustrated in the case of the $e^+e^- \to ZH$ and $\mu^+\mu^- \to H$ production processes for a two-doublet Higgs model with CP-violating neutral sector. A very useful technique for minimizing 'depolarization-factor' suppressions of the correlations in the $t\bar{t}$ mode is introduced.

Determination of the CP nature of any neutral Higgs boson that is directly observed will be crucial to fully unravelling the nature of the Higgs sector. The Standard Model (SM) Higgs boson is CP-even, while the Minimal Supersymmetric Model (MSSM) predicts two pure CP-even and one pure CP-odd neutral Higgs boson. More generally, it is entirely possible to have either explicit or spontaneous CP violation in the neutral Higgs sector. Indeed, the simplest non-supersymmetric two-Higgs-doublet model (2HDM) and the supersymmetric Higgs two-doublet plus singlet model both allow for Higgs mass eigenstates of impure CP nature.[1] Here we shall focus on the 2HDM, in which CP violation results in three neutral states, $H_{i=1,2,3}$, of mixed CP character.

Various types of experimental observables can be considered for determining the CP character of a given Higgs boson. The most direct probe is provided by comparing the Higgs boson production rate in collisions of two back-scattered-laser-beam photons of various different polarizations.[2] A certain difference in rates for different photon helicity choices is non-zero only if CP violation is present, and has a good chance of being of measurable size for many 2HDM parameter choices. In the case of a CP-conserving Higgs sector, the dependence of the $\gamma\gamma \to H$ cross section on the relative orientation of the transverse polarizations of the two colliding photons may well allow a determination of whether a given H is CP-even or CP-odd[3,4] Note that the general utility of the photon-photon collision polarization asymmetries derives from the fact that the (one-loop) $\gamma\gamma$ couplings

of a CP-odd and CP-even H are similar in magnitude, so that sensitivity is not strongly dependent upon the CP nature of the H.

Correlations between decay products can also probe the CP nature of a Higgs boson. In this paper we focus on effects that arise entirely at tree-level, *i.e.* that do not rely on imaginary parts generated at one-loop. For the dominant two-body decays of a Higgs boson, we can define appropriate observables if we are able to determine the rest frame of the Higgs boson and if the secondary decays of the primary final state particles allow an analysis of their spin or helicity directions. An obvious example is to employ correlations between the decay planes of the decay products of WW or ZZ vector boson pairs and/or energy correlations among the decay products.[5-11] However, these will not be useful for a purely CP-odd H (which has zero tree-level WW, ZZ coupling and thus decays primarily to $F\overline{F}$) or for a mixed-CP H in the (most probable) case where the CP-even component accounts for essentially all of the WW, ZZ coupling strength (thereby yielding 'apparently CP-even' correlations). In contrast, H decays to $\tau^-\tau^+$ or $t\bar{t}$, followed by τ or t decays, do, in principle, allow equal sensitivity to the CP-even and CP-odd components of a given Higgs boson.

Indeed, a H eigenstate couples to $F\overline{F}$ according to: $\mathcal{L} \propto \overline{F}(a+ib\gamma_5)FH$,which yields

$$\langle F_+\overline{F}_+|H\rangle \propto b + ia\beta_F; \quad \langle F_-\overline{F}_-|H\rangle \propto b - ia\beta_F , \tag{1}$$

where $\beta_F = \sqrt{1 - 4m_F^2/m_H^2}$, helicity-flip amplitudes being zero. The crucial point is that, in general, a and b are of comparable magnitude in a CP-violating 2HDM. In the notation of Ref. [12] we have

$$
\begin{aligned}
a_{t\bar{t}} &= -\frac{m_t s_1 c_3}{v\sin\beta}, & b_{t\bar{t}} &= -\frac{m_t s_1 s_3 \cos\beta}{v\sin\beta}, \\
a_{\tau^-\tau^+} &= -\frac{m_\tau c_1}{v\cos\beta}, & b_{\tau^-\tau^+} &= -\frac{m_\tau s_1 s_3 \sin\beta}{v\cos\beta},
\end{aligned}
\tag{2}
$$

where $v = 246$ GeV, $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values of the neutral Higgs fields that couple to up and down-type quarks, and c_1, s_1, c_3, s_3 are cosines and sines of neutral Higgs sector mixing angles α_1, α_3; the couplings of Eq. (2) are those that appear in the Euler angle parameterization of the first Higgs eigenstate as defined in Ref. [12]. In this same notation, the WW coupling of H is $c_1 \cos\beta + s_1 c_3 \sin\beta$ times the WW coupling of the SM Higgs boson. Note that we have assumed a Type-II (as defined in Ref. [1]) 2HDM model, in which charged-lepton couplings of the H are analogous to down-quark couplings. It is consistent to take H to be the lightest eigenstate and to allow the angles $\alpha_{1,3}$ appearing in

c_1, s_1, c_3, s_3 to take on arbitrary values.* Clearly, a and b are generally of similar size. SM couplings for the H are obtained in the case $\alpha_1 = \beta$ and $\alpha_3 = 0$.

The use of azimuthal angular correlations in $\tau^-\tau^+$ and $t\bar{t}$ decays to determine the CP eigenvalue of a pure CP state was explored in Ref. [4]; the additional azimuthal angle dependence that is present only for a mixed-CP eigenstate has been noted in Ref. [8] in a special case of $t\bar{t}$ decays. Here, we shall present a unified treatment aimed at realistically evaluating the possibility of using correlations in $H \to \tau^-\tau^+$ and $t\bar{t}$ final states to determine if a decaying Higgs boson is a mixed CP eigenstate, thereby directly probing for the presence of CP violation in the Higgs sector.

An efficient framework for our analysis is that developed in Refs. [13,4]. Consider the charged current decay $F \to Rf$, where F is a heavy fermion, f is a light fermion whose mass can be neglected, and R can be either a single particle or a multiparticle state with known quantum numbers and, therefore, calculable coupling to the charged weak current. (Examples are $\tau \to R\nu$, where $R = \pi, \rho, A_1, \ldots$, and $t \to Wb$, where W decays to a fermion plus anti-fermion.) For the R's of interest, the form of the hadronic current J_μ, deriving from the standard $V - A$ interaction for the $J_\mu \equiv \langle R|V_\mu - A_\mu|0\rangle$ coupling, is completely determined in terms of the final particle momenta. Using the particle symbol to denote also its momentum, and defining

$$\Pi_\mu = 4\mathrm{Re}\, J_\mu f \cdot J^* - 2f_\mu J \cdot J^*, \quad \Pi_\mu^5 = 2\epsilon_{\mu\rho\nu\sigma}\mathrm{Im}\, J^\rho J^{*\,\nu} f^\sigma, \tag{3}$$

all useful correlations in $H \to F\overline{F}$ decay can be obtained by employing the quantities

$$\omega = F \cdot (\Pi - \Pi^5), \quad R_\mu = m_F^2(\Pi - \Pi^5)_\mu - F_\mu F \cdot (\Pi - \Pi^5), \tag{4}$$

and their $\overline{F}$ analogues. In the F rest frame, $R_0 = 0$, $\vec{R} = m_F^2(\vec{\Pi} - \vec{\Pi}^5)$, and $|\vec{R}| = m_F\omega$. In fact, $\vec{S}_F = \vec{R}/(m_F\omega)$ acts as an effective spin direction ($|\vec{S}_F|^2 = 1$) when in the F rest frame.

Let us give some illustrative examples. For $\tau^- \to \pi^-\nu$ decay, $J_\mu \propto \pi_\mu^-$ and $\vec{S}_{\tau^-} = \hat{\pi^-}$ is the unit vector pointing in the direction of the π^-'s three momentum (using angles defined in the τ^- rest frame). For $\tau^- \to \rho^-\nu \to \pi^-\pi^0\nu$, $J_\mu \propto (\pi^- - \pi^0)_\mu$, yielding $\Pi_\mu \propto 4(\pi^- - \pi^0)_\mu\nu \cdot (\pi^- - \pi^0) + 2\nu_\mu m_\rho^2$, and, thence, $\vec{S}_F \propto m_\tau(\vec{\pi^-} - \vec{\pi^0})(E_{\pi^-} - E_{\pi^0}) + \vec{\nu}m_\rho^2/2$, where the pion energies and directions are defined in the τ^- rest frame. For $t \to W^+b \to l^+\nu b$, $J_\mu \propto \overline{u}(\nu)\gamma_\mu(1 - \gamma_5)v(l^+)$,

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$\star$ For fixed $\tan\beta$ and v, there are 7 independent parameters in the 2HDM model in which the discrete symmetry that guarantees the absence of flavor-changing neutral currents is only softly-broken.[12] These can be taken as α_i ($i = 1, 2, 3$), the masses of the three neutral Higgs bosons, and the mass of the charged Higgs boson.

and $\Pi_\mu \propto l_\mu^+ \nu \cdot b + \nu_\mu l^+ \cdot b$, $\Pi_\mu^5 \propto \nu_\mu l^+ \cdot b - l_\mu^+ \nu \cdot b$, so that $\Pi_\mu - \Pi_\mu^5 \propto l_\mu^+$, implying $\vec{S}_t = \hat{l}^+$ in the t rest frame.

If the full $(\Pi - \Pi^5)_\mu$ can be determined on an event-by-event basis, then we can define the 'effective spin' vectors $\vec{S}_F$ and $\vec{S}_{\overline{F}}$ *for each event*, and the distribution of the Higgs decay products takes the very general form

$$dN \propto \Big[(b^2 + a^2\beta_F^2)(1 + \cos\theta\cos\overline{\theta}) + (b^2 - a^2\beta_F^2)\sin\theta\sin\overline{\theta}\cos(\phi - \overline{\phi}) \tag{5}$$
$$- 2ab\beta_F \sin\theta\sin\overline{\theta}\sin(\phi - \overline{\phi}) \Big] d\cos\theta\, d\cos\overline{\theta}\, d\phi\, d\overline{\phi}\,,$$

where θ, ϕ and $\overline{\theta}, \overline{\phi}$ define the angles of $\vec{S}_F$ and $\vec{S}_{\overline{F}}$ in the F and $\overline{F}$ rest frames, respectively, *employing the direction of F in the H rest frame as the coordinate-system-defining z axis.*[$\star$] (Note that the F coordinate axes are to be used in both the F *and* $\overline{F}$ rest frames to define the angles appearing in Eq. (5).) To employ Eq. (5) we must retain the ability to distinguish F from $\overline{F}$. In fact, to determine $\sin(\phi - \overline{\phi})$ it is absolutely necessary that we be able to determine the F and $\overline{F}$ rest frames, *i.e.* their line of flight in the Higgs rest frame. We shall return to this issue momentarily.

We note that at one-loop a and b can develop imaginary parts. In this case, a^2, b^2, ab should be replaced in Eq. (5) by $|a|^2, |b|^2, \mathrm{Re}(ab^*)$, respectively. In addition, new angular dependences arise in Eq. (5) of the form $-2\mathrm{Im}(ab^*)\beta_F(\cos\theta + \cos\overline{\theta})$. In principle, the imaginary parts of a and b are also sensitive to CP violation in the Higgs sector (but also to other types of CP violation — in the SM, non-zero effects appear at the 2-loop level). After including branching ratios, the statistical significance associated with isolating the above term is not very large if the 2HDM provides the only source of CP violation.[14] In any case, these extra terms will not contribute to the correlations upon which we focus.

If we cannot determine $(\Pi - \Pi^5)_\mu$ for each event, then Eq. (5) must be modified. An extreme example is $F \to Rf$ decay where the R decay products are not examined. In this case the angles of R in the F rest frame would be employed in Eq. (5), and 'depolarization' factors arise as a result of event averaging. In deriving Eq. (5), the angular independent term is actually multiplied by $(m_F\omega_F)(m_F\omega_{\overline{F}})$ and the $\cos\theta\cos\overline{\theta}$, $\sin\theta\sin\overline{\theta}\sin(\phi - \overline{\phi})$ and $\sin\theta\sin\overline{\theta}\cos(\phi - \overline{\phi})$ terms by $|\vec{R}_F||\vec{R}_{\overline{F}}|$. On an event-by-event basis the ratio of these coefficients is unity, as outlined earlier. When averaged over events, this is no longer true. Consequently, when event

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[$\star$] Defining the 4-vector $S_\mu = R_\mu/(\omega m_F)$, and similarly for $\overline{S}_\mu$, the underlying covariant form of the matrix element squared is $|\mathcal{M}|^2 \propto (a^2 + b^2)(F \cdot \overline{F} - m_F^2 S \cdot \overline{S}) + (a^2 - b^2)(F \cdot \overline{F} S \cdot \overline{S} - \overline{F} \cdot S F \cdot \overline{S} - m_F^2) - 2ab\epsilon_{\alpha\beta\rho\sigma}S^\alpha \overline{S}^\beta F^\rho \overline{F}^\sigma$, with the convention $\epsilon_{0123} = +1$. We note that this result reduces to that of Ref. [8] (except for a difference in the sign of the ϵ term) in the case of $t\overline{t}$ where (using their notation $\overline{l} \equiv l^+$) $S_\mu = (m_F\overline{l}_\mu/F \cdot \overline{l} - F_\mu/m_F)$.

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averaging (denoted by $\langle\ldots\rangle$) all the angle-dependent terms in Eq. (5) must be multiplied by $D_F \equiv \langle|\vec{R}_F|\rangle/(m_F\langle\omega_F\rangle)$ and/or its $D_{\overline{F}}$ analogue, relative to the angle-independent term. We define $D \equiv D_F D_{\overline{F}}$.

At first sight, the necessity of event averaging arises in the case of the $t\overline{t}$ final state, for which we will find that we must have one top decay leptonically and the other hadronically in order to define the $t\overline{t}$ line of flight and, thereby, appropriate angles in Eq. (5). For the hadronically decaying top, the problem is to distinguish the quark vs. anti-quark jet coming from the W so as to construct $(\Pi - \Pi^5)_\mu$ (which is proportional to the W^+ (W^-) anti-quark (quark) momentum for t ($\overline{t}$) decay) for each event.[†] If we simply sum over all W decay product configurations, then the appropriate depolarization factor is easily computed by using $J_\mu \propto \epsilon_\mu^W$ and summing over W polarizations. One finds $\Pi_\mu^5 = 0$ and $\Pi_\mu \propto b_\mu + W_\mu(m_t^2 - m_W^2)/m_W^2$. Employing this result yields $D_t = (m_t^2 - 2m_W^2)/(m_t^2 + 2m_W^2) \sim 0.4$ for $m_t = 174$ GeV. In the modified Eq. (5) the angles for the one hadronically decaying t (or $\overline{t}$) would then be those of the W^+ (or W^-) in the t (or $\overline{t}$) rest frame. (For the leptonically decaying $\overline{t}$ (or t) the angles of the l^- (or l^+) are directly measured and the associated $D_{\overline{t}}$ (or D_t) is unity.) Similarly, if in $\tau \to R\nu$ the R is spin-1 and its decay products were simply integrated over, a depolarization factor of $D_\tau = (m_\tau^2 - 2m_R^2)/(m_\tau^2 + 2m_R^2)$ would enter.

Fortunately, these severe depolarization factors can be avoided in both cases. For the bulk of τ decays R is a resonance of known quantum numbers decaying to easily distinguished particles, in which case we can construct $(\Pi - \Pi^5)_\mu$ event-by-event (see, $e.g.$, the ρ example described earlier) and depolarization factors do not arise. In the case of a hadronically decaying t ($\overline{t}$), a simple helicity argument shows that the most energetic of the W^+ (W^-) jets in the t ($\overline{t}$) rest frame is more likely to be the anti-quark (quark), $i.e.$ the equivalent of the l^+ (l^-). Employing the angles of this most energetic jet (while integrating over the angles of all the other jets, so that the angles of this most energetic jet define the only direction associated with the decay) yields (via Monte Carlo calculation) $D \sim 0.78$, essentially independent of m_H for $m_H \lesssim 800$ GeV.

Let us now specify our procedure for isolating the coefficients of the $\cos(\phi - \overline{\phi})$ and $\sin(\phi - \overline{\phi})$ angular correlation terms. Defining $c \equiv \cos\theta$, $\overline{c} \equiv \cos\overline{\theta}$, $s \equiv \sin\theta$, $\overline{s} \equiv \sin\overline{\theta}$, $c_\phi \equiv \cos\delta\phi$, $s_\phi \equiv \sin\delta\phi$ (where $\delta\phi \equiv \overline{\phi} - \phi$), and $d\Omega \equiv dc\,d\overline{c}\,d\delta\phi$, and including a possible depolarization factor, we have

$$\frac{1}{N}\frac{dN}{d\Omega} = \frac{1}{8\pi}\left[1 + Dc\overline{c} + \rho_1 s\overline{s}s_\phi + \rho_2 s\overline{s}c_\phi\right] , \tag{6}$$

[†] We assume that there is no efficient technique for determining the sign of the charges of the quark jets resulting from the top decay.

where

$$\rho_1 \equiv D\frac{2ab\beta_F}{(b^2 + a^2\beta_F^2)}, \quad \rho_2 \equiv D\frac{(b^2 - a^2\beta_F^2)}{(b^2 + a^2\beta_F^2)}. \tag{7}$$

For a CP-conserving Higgs sector, either $a = 0$ or $b = 0$ implying $\rho_1 = 0$ and $|\rho_2| = D$. For a CP-mixed eigenstate, both a and b are non-zero. Thus $\rho_1 \neq 0$ provides an unequivocable signature for CP violation in the Higgs sector, while the difference $D - |\rho_2|$ also provides a measure of Higgs sector CP violation. (Indeed, ρ_1 and ρ_2 are not independent; $\rho_1^2 + \rho_2^2 = D^2$.) Values of $\rho_1 \sim D$ and $\rho_2 \sim 0$ are common in an unconstrained 2HDM.

To isolate ρ_1 and ρ_2, we define projection functions $f_{1,2}(\theta, \overline{\theta}, \delta\phi)$ such that $\int f_{1,2}d\Omega = 0$, $\int f_{1,2}c\overline{c}d\Omega = 0$, $\int f_1 s\overline{s}s_\phi d\Omega = 8\pi$, $\int f_1 s\overline{s}c_\phi d\Omega = 0$, $\int f_2 s\overline{s}s_\phi d\Omega = 0$, and $\int f_2 s\overline{s}c_\phi d\Omega = 8\pi$. Then, $\rho_{1,2} = \int f_{1,2}\frac{1}{N}\frac{dN}{d\Omega}d\Omega$. The critical question is with what accuracy can $\rho_{1,2}$ be determined experimentally? In the absence of background, it is easily shown that the experimental errors of the determination are given by $\delta\rho_{1,2} = (y_{1,2} - \rho_{1,2}^2)^{1/2}/\sqrt{N}$, where $y_{1,2} = \int f_{1,2}^2\frac{1}{N}\frac{dN}{d\Omega}d\Omega$, and N is the total number of events. (For the $f_{1,2}$ choices we shall make, $y_{1,2} = \int f_{1,2}^2 d\Omega/(8\pi)$.) If background is present, then this result is modified to $\delta\rho_{1,2} = [y_{1,2} - \rho_{1,2}^2 + (B/S)(y_{1,2} + \rho_{1,2}^2 - 2\rho_{1,2}\rho_{1,2}^B)]^{1/2}/\sqrt{S}$, where S is the number of H events, B is the number of background events, $\rho_{1,2}^B$ is that for the background alone, and $\rho_{1,2}$ refers to the signal only. This result assumes that the background is precisely known, either by detector Monte Carlo plus theory or high precision experimental measurement. The choices $f_1 = (8/\pi)\epsilon(s_\phi)$ and $f_2 = (8/\pi)\epsilon(c_\phi)$ [where $\epsilon(a) = +1$ (-1) for $a > 0$ $(a < 0)$] are equivalent to employing simple asymmetries, and yield (for $B = 0$) $y_{1,2} = (8/\pi)^2$ and $\rho_{1,2}/\delta\rho_{1,2} = (\pi\rho_{1,2}/8)\sqrt{N}/[1 - (\pi\rho_{1,2}/8)^2]^{1/2}$. (Note that $(\pi/8)\rho_{1,2}$ are the magnitudes of the $(N_+ - N_-)/(N_+ + N_-)$-type asymmetries.) For a functional form expressed in terms of orthogonal functions (upon integration over $d\Omega$), the error is minimized by using projection functions which match the angular dependence of the term of interest. Thus, we employ $f_1 = (9/2)s\overline{s}s_\phi$ and $f_2 = (9/2)s\overline{s}c_\phi$, for which $y_{1,2} = 9/2$ and $\rho_{1,2}/\delta\rho_{1,2} = \sqrt{2/9}\rho_{1,2}\sqrt{N}/[1 - (2/9)\rho_{1,2}^2]^{1/2}$. Note that $\sqrt{2/9} \sim 0.47 > (\pi/8) \sim 0.39$.

We now discuss the Higgs production reactions and the $\tau^-\tau^+$ and $t\overline{t}$ final state decay modes for which the angles of Eq. (5) can be experimentally determined. Consider first the $\tau^-\tau^+$ case. The τ decays are of two basic types: $\tau \to l\nu\nu$ and $\tau \to R\nu$, where R is a hadronically decaying resonance of known quantum numbers. Together these constitute about 95% of the τ decays, with $BR(\tau \to \Sigma R\nu) \sim 58.8\%$.[15] In the presence of two or more neutrinos, we cannot determine the τ rest frame angles without employing a Higgs production reaction in which the Higgs rest frame can be determined without reference to its decays. Even if we know the Higgs rest frame, if either (or both) τ's decay leptonically we will still not be able to determine either $\cos\delta\phi$ or $\sin\delta\phi$. Only by knowing the

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Higgs rest frame and having both τ's decay to $R\nu$ are there enough constraints to unambiguously determine $|\delta\phi|$.[4] For such decays it is crucial that the charge of R can be determined from an examination of its decay products; in fact we assume that the R decay products can be fully identified so that $(\Pi - \Pi^5)_\mu$ can be determined for each event, thereby avoiding any depolarization factor. Thus, we employ $D = 1$ and an effective branching ratio for useful $\tau^-\tau^+$ final states of $(0.588)^2$. Because of a two-fold ambiguity in the kinematic solution, determination of the sign of $\delta\phi$ generally requires vertex tagging of both the τ's.[16] Thus, the $(0.588)^2$ should also be multiplied by the square of the efficiency for vertex tagging a τ when estimating our ability to measure ρ_1; however, because this efficiency is strongly detector-dependent we do not include it in our explicit numerical results for ρ_1.

In the case of $t\bar{t}$ decays, we cannot employ purely hadronic final states for which we would be unable to distinguish t from $\bar{t}$. Even if the Higgs rest frame is known (and the four-momenta of both b-jets are measured), double leptonic decays lead to a two-fold ambiguity in the determination of $\delta\phi$, so that only $\cos\delta\phi$ could be computed.[⋆] (As we see from Eq. (6), this is adequate for ρ_2.) Only in the case where one top decays hadronically, and the other leptonically, are we simultaneously able to determine the exact $t\bar{t}$ decay axis and distinguish t from $\bar{t}$. Thus, we employ an effective branching ratio for useful $t\bar{t}$ final states of $2 \times (2/3) \times (2/9)$ (keeping only $l = e, \mu$). As noted earlier, employing *one* hadronic t (or $\bar{t}$) decay and identifying the most energetic jet from the W^+ (W^-) with the anti-quark (quark) leads to a depolarization factor of $D \sim 0.78$.[†] Finally, we note that for a known Higgs mass and known top mass, the kinematical constraints in the single-leptonic $t\bar{t}$ final states are sufficient to determine unambiguously the momentum of the single ν, without knowing ahead of time the Higgs rest frame. This implies that the $t\bar{t}$ final state correlations could in principle be employed at a hadron collider, although the extra initial and final state radiation present in hadronic collisions is very likely to lead to too much confusion for this to work in practice.

In order to assess our ability to experimentally measure ρ_1 and ρ_2, we have examined H production in the reactions $e^+e^- \to ZH$ at a future linear e^+e^- collider and $\mu^+\mu^- \to H$ at a possible future $\mu^+\mu^-$ collider.[17] (In the ZH reaction, we employ both hadronic and $l = e, \mu$ charged-leptonic decay modes for the Z, with net total branching ratio of $\sim 76\%$.) For both reactions, the Higgs rest frame can be determined without reference to the H decays. For the e^+e^- collider we have adopted the optimal energy, $\sqrt{s} = m_Z + \sqrt{2}m_H$, as a function of Higgs mass, and

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⋆ The $\sin\delta\phi$ dependence involving just the two charged leptons in the double leptonic $t\bar{t}$ final state pointed out in Ref. 8 thus cannot be experimentally isolated.

† If the Higgs rest frame is known, ρ_2 can be obtained for double-leptonic decays with branching ratio $(2/9)^2$ and $D = 1$. Combining this channel with the semi-leptonic channel would result in roughly a 13% increase of the statistical significance values for a CP violation signal that we shall present in the case of ρ_2.

assumed an integrated luminosity of 85 fb^{-1}. Our results for statistical significances will assume that this ZH mode is essentially background free. However, we have not incorporated any efficiencies for cuts that might be required to guarantee this. We have also not included the dilution due to the ZZ continuum background for m_H values in the vicinity of m_Z. Depending upon the detector resolution, this background can be substantial for m_H values between about 75 and 105 GeV. In this interval, our results should (at best) be considered an upper bound. For the $\mu^+\mu^-$ collider we have computed the Higgs signal and the continuum $\tau^-\tau^+$ and $t\bar{t}$ backgrounds assuming unpolarized beams and a machine energy resolution of 0.1%, with $\sqrt{s}$ centered at the (already known) value of m_H. We adopt an integrated luminosity of 20 fb^{-1}. This is consistent with the hoped for integrated luminosity of $100 - 200$ fb^{-1} of a multi-TeV $\mu^+\mu^-$ collider when run at the lower energies required for direct production of the H in the mass range considered.

For m_H values such that the $e^+e^- \rightarrow ZH$ production mode is background free, the statistical significance of a non-zero result for ρ_1 is that given earlier, $N_{SD}^1 = |\rho_1|/\delta\rho_1$, where $\delta\rho_1 = (9/2 - \rho_1^2)^{1/2}/\sqrt{N}$, and N is the number of events *after including the branching ratios required to achieve the final state of interest*: $BR_{eff} = BR(H \rightarrow F\overline{F}) \times BR(F\overline{F} \rightarrow X)$, where the latter $F\overline{F}$ branching ratios to useful final X states were specified above. In the case of ρ_2 we must actually determine the statistical significance associated with a measurement of $D - |\rho_2|$. This is given by $N_{SD}^2 = [D - |\rho_2|]/\delta\rho_2$, where $\delta\rho_2 = (9/2 - \rho_2^2)^{1/2}/\sqrt{N}$.

In $\mu^+\mu^- \rightarrow H$, the continuum backgrounds must be included. The CP-conserving background does not contribute to ρ_1, and the statistical significance of a non-zero value for ρ_1 is given by $N_{SD}^1 = |\rho_1|/\delta\rho_1$ with $\delta\rho_1 = [9/2 - \rho_1^2 + (B/S)(9/2 + \rho_1^2)]^{1/2}/\sqrt{S}$, where S is the total number of events from H production, and B is the total number of events from the continuum background, in the final state of interest. Although the background does have substantial $\cos\delta\phi$ dependence, not only should we have an excellent theory plus detector Monte Carlo simulation of the background, but also ρ_2 for the background could be directly measured for $\sqrt{s}$ values on either side of the Higgs resonance. Thus, we neglect errors in the background subtraction. In this case, we have $N_{SD}^2 = [D - |\rho_2|]/\delta\rho_2$ with $\delta\rho_2 = [9/2 - \rho_2^2 + (B/S)(9/2 + \rho_2^2 - 2\rho_2\rho_2^B)]^{1/2}/\sqrt{S}$, where ρ_2^B is that for the background alone. (The non-superscripted $\rho_{1,2}$ are always those of the signal alone.) Results presented for N_{SD}^2 include ρ_2^B, but differ negligibly from those obtained if ρ_2^B is set to zero in the error expression above.

Our results for the maximum N_{SD}^1 and N_{SD}^2 values are presented in Fig. 1, where we have adopted a top quark mass of 174 GeV. The maximum values were found by searching over all values for the Euler angles α_1 and α_3 appearing in Eq. (2), holding $\tan\beta$ and m_H fixed. In general, N_{SD}^1 is only slightly larger than N_{SD}^2, as is easy to understand from the fact that $|\rho_1|$ and $D - |\rho_2|$ both have maximum values close to D. The production rates and branching ratios both depend upon the couplings of Eq. (2), as well as couplings to other fermions and to WW and ZZ pairs. Couplings to up-type fermions are, of course, analogous to the

Figure 1: The maximum statistical significances N_{SD}^1 and N_{SD}^2 for $H \to \tau^-\tau^+$ (————) and $H \to t\bar{t}$ ($-\ -\ -\ -$), in $e^+e^- \to ZH$ ($L = 85$ fb^{-1}) and $\mu^+\mu^- \to H$ ($L = 20$ fb^{-1}) production, after searching over all α_1 and α_3 values at fixed m_H and $\tan\beta$. In each case, curves for the three $\tan\beta$ values of 0.5, 2, and 20 are shown. In the $\tau^-\tau^+$ ($t\bar{t}$) mode N_{SD} values increase (decrease) with increasing $\tan\beta$, except in the case of $\mu^+\mu^- \to H \to t\bar{t}$, where the lowest curve is for $\tan\beta = 0.5$, the highest curve is for $\tan\beta = 2$, and the middle curve is for $\tan\beta = 20$.

$t\bar{t}$ coupling given in Eq. (2) with m_t replaced by the up-type fermion mass, while couplings to down-type fermions are analogous to the $\tau^-\tau^+$ coupling of Eq. (2) with m_τ replaced by the down-type fermion mass. In computing the $\tau^-\tau^+$ and $t\bar{t}$ branching ratios, the full set of possible H decays to $f\bar{f}$, WW, and ZZ are included. As noted earlier, the results of Fig. 1 for N_{SD}^1 in the $\tau^-\tau^+$ mode do not incorporate efficiencies for τ vertex tagging, required to determine the sign of $\delta\phi$ (as needed for computing ρ_1), and thus should be multiplied by the efficiency (*not* its square) for τ tagging. Fortunately, it is expected that this efficiency will be relatively high for appropriate detector designs.

Consider first the results for $e^+e^- \to ZH$ collisions. From Fig. 1 we find that detection of CP violation through both ρ_1 and ρ_2 is very likely to be possible for $m_H < 2m_W$ via the $H \to \tau^-\tau^+$ decay mode. This is an important result given that various theoretical prejudices suggest that the lightest Higgs boson is quite likely to be found in this mass range. For m_H between $2m_W$ and $2m_t$, a statistically significant measurement of CP violation will be difficult. For $m_H > 2m_t$, detecting CP violation in the $t\bar{t}$ mode would require a somewhat larger L (of order 5 times

the assumed luminosity of $L = 85$ fb^{-1} for $\tan\beta$ between 2 and 5).

In $\mu^+\mu^- \to H$ production, Fig. 1 shows that the maximum N^1_{SD} and N^2_{SD} values in the $\tau^-\tau^+$ mode can remain large out to large Higgs masses if $\tan\beta$ is large, but that for small to moderate $\tan\beta$ values the statistical significances are better in $e^+e^- \to ZH$ collisions when $m_H < 2m_W$. The reason for this is obvious — Higgs production in $\mu^+\mu^-$ collisions is strongly enhanced at large $\tan\beta$. However, Fig. 1 also indicates that the $\mu^+\mu^- \to H \to \tau^-\tau^+$ channel has the advantage of possibly small sensitivity to the WW decay threshold at $m_H \sim 2m_W$. Such insensitivity arises when the Euler angles α_1, α_3 are chosen so as to minimize WW, ZZ couplings (and hence $H \to WW, ZZ$ branching ratios) without sacrificing production rate. Thus, for $L = 20$ fb^{-1} $\mu^+\mu^-$ collisions could allow detection of CP violation all the way out to $2m_t$ for $\tan\beta \gtrsim 10$. The $t\bar{t}$ final state extends the range of m_H for which detection of CP violation might be possible only somewhat, and only if $\tan\beta$ lies in the moderate range near 2. A factor 10 higher $\mu^+\mu^-$ luminosity (*i.e.* $L = 200$ fb^{-1}, requiring a machine design focusing on center-of-mass energies below 1 TeV and, possibly, several years of running) would extend the range of possible detection in both modes: for $\tan\beta \gtrsim 2$, m_H values up to $2m_t$ could be probed in the $\tau^-\tau^+$ mode, while the $t\bar{t}$ mode might be useful out to quite high masses.

Of course, in obtaining the above results we have implicitly assumed that the H does not have additional decays. If it is not the lightest Higgs eigenstate, decays of the H to a pair of lighter Higgs bosons or to Z plus a lighter Higgs boson might be kinematically allowed. If present, they would dilute the statistical significances of Fig. 1. However, if decays involving other Higgs eigenstates are significant, then there are many other direct 'signals' of CP violation in the Higgs sector that could be present. For example, simultaneous presence of $H_2 \to H_1 H_1$ and either $H_2 \to ZH_1$ decays (in our notation, H_2 is the heavier state) or $e^+e^- \to Z \to H_1 H_2$ production at a significant level would alone require the H's to be a mixture of CP-even (allowing decays to a pair of Higgs) and CP-odd (allowing Z plus Higgs decay) states. As another example, a substantial production rate for $H_{1,2}$ in $e^+e^- \to ZH_{1,2}$ combined with either the existence of $H_2 \to ZH_1$ decays or $e^+e^- \to Z \to H_1 H_2$ production would imply that the couplings ZZH_1, ZZH_2 and $ZH_1 H_2$ are all non-zero, which requires CP violation in the Higgs sector.[18]*

We have not explicitly analyzed the case of a (non-minimal) supersymmetric model with CP violation in the Higgs sector. However, several comments are useful. First, decays to superpartner pairs (neutralino, chargino, slepton, squark pairs) might be important and would dilute the observability of ρ_1 and ρ_2 in the $\tau^-\tau^+$ and $t\bar{t}$ channels. In this case, one can consider using the the supersymmetric

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⋆ Strictly speaking, the above statements are only true with regard to tree-level couplings; a ZZH coupling is present at one-loop even if the H is purely CP-odd. To completely avoid contamination from C-violating one-loop diagrams, three or more neutral Higgs bosons must be detected. For example, to all orders, non-zero values for all three of the couplings $ZH_1 H_2$, $ZH_1 H_3$ and $ZH_2 H_3$ are only possible if CP violation is present.

partner pair events themselves. Generally, a measurement of $\rho_{1,2}$ in superparticle pair channels would probe a subtle mixture of Higgs sector CP violation and CP violation deriving from complex phases in the soft-supersymmetry-breaking parameters that enter the chargino, neutralino, *etc.* mass matrices. However, restrictions from neutron and electron electric dipole moments suggest that the latter phases are quite small, in which case an observable non-zero value for ρ_1 or $D - |\rho_2|$ would imply CP-violation in the Higgs sector. Procedurally, those events in which each member of the superparticle pair decays to jets and/or charged leptons plus a single lightest neutralino (*i.e.* the LSP, of presumably known mass) would allow treatment along the same lines as the $\tau^- \tau^+$ mode after correcting for the finite mass of the single invisible LSP. Generally, lifetimes would be too short for vertex tagging, and only $|\delta\phi|$ could be determined, so that only measurement of ρ_2 would be possible. Of course, if the decays of a supersymmetric model H were spread out over many channels, there might be inadequate statistics in any one channel.

It is also useful to comment on how well ρ_1 and ρ_2 can be measured in the case of a CP-conserving Higgs sector. Recall that $\rho_1 = 0$ and $\rho_2 = +D, -D$ for a CP-odd, CP-even Higgs boson. As an example, consider $e^+ e^- \to ZH$, where H is CP-even. (Only a CP-even Higgs boson will have usable $e^+ e^- \to ZH$ production rate.) For simplicity, we assume that the H has SM couplings. (The results will then also apply to the lightest CP-even MSSM Higgs boson h^0 in the case where the other Higgs bosons have masses $m_{A^0} \sim m_{H^0} \sim m_{H^\pm} \gtrsim 2m_Z$, in which limit the h^0 is SM-like.) For $L = 85$ fb^{-1}, we find that the $\tau^- \tau^+$ mode yields $\delta\rho_1 \sim \delta\rho_2$ increasing from ~ 0.05 to ~ 0.13 as m_H ranges from 60 GeV up to 160 GeV. Comparing to 1 ($D = 1$ in this case), we see that a simultaneous measurement of ρ_1 and ρ_2 would provide a very strong confirmation that a SM-like Higgs boson is indeed CP-even. ρ_1 would be particularly valuable for $m_H \sim m_Z$ since the ZZ continuum background yields non-trivial $\cos\delta\phi$ dependence. For $m_H > 2m_W$, even a rough determination of $\rho_{1,2}$ would not be possible in this mode due to the rapid fall in the $H \to \tau^- \tau^+$ branching ratio resulting from the onset of WW and ZZ decays. In the $t\bar{t}$ mode, $\delta\rho_1 \sim \delta\rho_2$ ranges between 0.4 and 0.8 for m_H between $2m_t$ and 750 GeV. Comparing to $D \sim 0.78$ we see that statistics would be inadequate to clearly distinguish between a SM-like CP-even Higgs boson and one of mixed-CP nature on the basis of the azimuthal angle correlations. Of course, in a general 2HDM, the H in question might be CP-even but have reduced WW, ZZ coupling. (In this case, the reduced production rate via $e^+ e^- \to ZH$ would be apparent, but would not on its own indicate whether the Higgs was CP-even or of mixed-CP character.) For reduced WW, ZZ coupling, the $H \to WW, ZZ$ branching fractions decline much more rapidly than the ZH cross section and the errors on ρ_1 and ρ_2 in the $t\bar{t}$ mode can be sufficiently small to provide a strong indication of the CP character of the H.

In summary, our results show that if the Higgs sector is CP-violating then there is a substantial possibility of explicitly exposing this CP violation through azimuthal angle correlations between final state particles in $H \to \tau^- \tau^+$, where the H is produced via $e^+ e^- \to ZH$ or $\mu^+ \mu^- \to H$, with assumed integrated luminosities of $L = 85$ fb^{-1} and $L = 20$ fb^{-1}, respectively. Of particular importance is the general utility of the $\tau^- \tau^+$ mode in $e^+ e^- \to ZH$ collisions for Higgs masses in the

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theoretically preferred $m_H \lesssim 2m_W$ region. For $m_H > 2m_t$, azimuthal correlations in the $t\bar{t}$ mode also provide sensitivity to CP violation. However, statistically reliable correlation measurements are predicted to be possible for a smaller portion of parameter space, which, however, would expand considerably if higher luminosities were available. The correlations employed rely on the fact that CP violation generally leads to non-trivial dependence (not present if the Higgs sector is CP-conserving) on the sine of an appropriately defined azimuthal angle ($\delta\phi$), and to dependence on the cosine of $\delta\phi$ that is substantially different than that predicted when CP is not violated (see Eqs. (5) and (6)). Of the two possible CP-violation-sensitive correlations, ρ_1 (obtained from the $\sin\delta\phi$ dependence) provides the best opportunity for detecting CP violation in the 2HDM Higgs sector, both because of a somewhat larger statistical significance, and because the associated non-trivial dependence on $\sin\delta\phi$ cannot arise from CP-conserving backgrounds or detector efficiency effects. However, ρ_2 (deriving from dependence on $\cos\delta\phi$) provides nearly as good a probe of CP violation. Further, there is a tendency for both CP violation 'signals' (namely a large value for ρ_1 and a small value for ρ_2) to be simultaneously substantial as a function of the two-Higgs-doublet model Higgs mixing angle parameters. On the experimental side, measurement of ρ_1 in the $\tau^-\tau^+$ mode requires high efficiency for τ vertex tagging, in order to determine the sign of the crucial $\delta\phi$ azimuthal angle, while prospects for measuring both ρ_1 and ρ_2 in the $t\bar{t}$ mode could be improved somewhat if a still more efficient technique for identifying the quark (anti-quark) jet in W^- (W^+) decay were available, e.g. by determining the charges of the jets coming from a hadronically decaying W.

Acknowledgements

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